Modeling of Electromagnetic Wave Interactions With Bi-isotropic and Bi-anisotropic Meta-materials Using A New Wavefield Decomposition Finite Difference Time Domain (WD-FDTD) Technique
PSU EM Research Group

- First group to apply wavefield decomposition techniques to FDTD
- First group to successfully develop dispersive FDTD formulation for general bi-anisotropic media (BIA-FDTD)
- First group to use new BIA-FDTD formulation to model electromagnetic wave interactions with dispersive chiral meta-materials
- Work has been recognized for its contribution to the EM modeling community
  - Received ACES Best Paper Award
  - IEEE Antennas and Propagation Society Conference Student Paper Competition Finalist
Subclasses of Bi-anisotropic Media

Bi-anisotropic media model

Anisotropy

Bi-isotropy

Dispersive

Chiral--$\varepsilon, \mu, \kappa$

Tellegen--$\varepsilon, \mu, \chi$
BI-FDTD Code

Chiral Model + Tellegen → General BI Media Model

General Dispersive BI Media Model + Dispersive ε, μ, κ, χ
BIA-FDTD Code

General BI Media Model + Anisotropy Effects

General BIA Media Model

General Dispersive BIA Media Model + Dispersive $\varepsilon, \mu, \kappa, \chi$
Objectives

- To derive a simple yet powerful way to model electromagnetic wave interactions with Bi-isotropic (BI) and Bi-anisotropic (BIA) media using the FDTD method

- To be able to extend the scheme to dispersive BI and BIA materials

- **Final product:** A very general and versatile FDTD code which can be used to model BIA media and its subclasses

- **Application:** A modeling tool for the analysis and design of microstrip antennas with BI and BIA substrates/superstrates
Bi-Anisotropic Media

\[ \mathbf{D} = \varepsilon \mathbf{E} + \zeta \mathbf{H} \]

\[ \mathbf{B} = \zeta \mathbf{E} + \mu \mathbf{H} \]

where \( \varepsilon, \xi, \zeta \) and \( \mu \) are tensor quantities in general
Field Components in FDTD

1-D

2-D

3-D

E

H
BI-FDTD Formulation

• Decomposing the electric and magnetic fields
  \[ \vec{E} = \vec{E}_+ + \vec{E}_- \]
  \[ \vec{H} = \vec{H}_+ + \vec{H}_- \]

• Wavefields: circularly polarized wave

• BI medium as an equivalent isotropic medium with \( \varepsilon_+ \), \( \mu_+ \) and \( \varepsilon_-, \mu_- \)

  \[ \mu_\pm = \mu(\cos \theta \pm \kappa_r)e^{\pm j\phi} \]
  \[ \varepsilon_\pm = \varepsilon(\cos \theta \pm \kappa_r)e^{\pm j\phi} \]
  \[ \cos \theta = \sqrt{1 - \chi_r^2} \]
BI-FDTD Formulation

- Wavefields are independent and uncoupled in a homogenous BI Medium

\[
\nabla \times E_\pm = -j\omega \mu_\pm H_\pm \\
\nabla \times H_\pm = j\omega \varepsilon_\pm E_\pm
\n\]

- \(E_\pm, H_\pm\) satisfy Maxwell’s equations in the equivalent media whenever \(E\) and \(H\) satisfy Maxwell equations in original BI medium
Test Case--Chiral Slab

Medium 1  Medium 2  Medium 3

\( \varepsilon_0, \mu_0 \)  \( \varepsilon, \mu_0, \kappa \)  \( \varepsilon_0, \mu_0 \)

Lattice Truncation (ABC)
Results

\[ \kappa_r = 0.003 \]
Anisotropic BI-FDTD

Equivalent Material Properties:

\[
\begin{bmatrix}
D_{x\pm} \\
D_{y\pm} \\
D_{z\pm}
\end{bmatrix} = \begin{pmatrix}
\varepsilon_{xx\pm} & \varepsilon_{xy\pm} & \varepsilon_{xz\pm} \\
\varepsilon_{yx\pm} & \varepsilon_{yy\pm} & \varepsilon_{yz\pm} \\
\varepsilon_{zx\pm} & \varepsilon_{zy\pm} & \varepsilon_{zz\pm}
\end{pmatrix}
\begin{bmatrix}
E_{x\pm} \\
E_{y\pm} \\
E_{z\pm}
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{x\pm} \\
B_{y\pm} \\
B_{z\pm}
\end{bmatrix} = \begin{pmatrix}
\mu_{xx\pm} & \mu_{xy\pm} & \mu_{xz\pm} \\
\mu_{yx\pm} & \mu_{yy\pm} & \mu_{yz\pm} \\
\mu_{zx\pm} & \mu_{zy\pm} & \mu_{zz\pm}
\end{pmatrix}
\begin{bmatrix}
H_{x\pm} \\
H_{y\pm} \\
H_{z\pm}
\end{bmatrix}
\]
Dispersive BI-FDTD

For the dispersive BI-FDTD formulation $\varepsilon$, $\mu$, and $\kappa$ are frequency dependant. The parameters $\varepsilon$ and $\mu$ use a Lorentzian model and for the chirality parameter, $\kappa$, a Condon Model is used.

\[
\varepsilon_\pm = \varepsilon(\omega) \left( 1 \pm \kappa_r(\omega) \right)
\]
\[
\mu_\pm = \mu(\omega) \left( 1 \pm \kappa_r(\omega) \right)
\]

\[
\varepsilon(\omega) = \varepsilon_o \left( \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty) \omega_o^2}{\omega_o^2 + j2\delta_e \omega - \omega^2} \right)
\]
\[
\mu(\omega) = \mu_o \left( \mu_\infty + \frac{(\mu_s - \mu_\infty) \omega_o^2}{\omega_o^2 + j2\delta_h \omega - \omega^2} \right)
\]
\[
\kappa(\omega) = \frac{j\omega \omega_o}{\omega_o^2 + j\delta_c \omega_o \omega - \omega^2}
\]
EM Wave Propagation in a Dispersive Chiral Half-Space

Position along z-axis within chiral medium (Cell No).

x-Component of the Electric Field (V/m)

y-Component of the Electric Field (V/m)
Dispersive Chiral Slab

This represents an example of a new left-handed meta-material that exhibits magneto-electric coupling.
Contributions

• The unique properties of BI and BIA media present many opportunities for the design of novel meta-materials

• Some applications include:
  – Loaded dielectric and ferrite antennas, plasma antennas, chirostrip antennas, BIA antennas
  – Left-handed or double-negative meta-materials
  – Coatings for scatterers
  – Depolarizers, modulators, sensors, etc…

• A new WD-FDTD approach is being developed at PSU to provide a powerful analysis tool to aid in the design of devices which incorporate BI and BIA meta-materials