Design of Metamaterial-Coated Arrays through Quasi-Conformal Transformation Optics

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***Abstract*— This paper presents synthesis of conformal arrays enhanced by metamaterial lenses. The synthesis uses a multi-step Quasi-Conformal Transformation Optics technique to compute lens parameters that allow an arbitrary-shaped conformal array to acquire properties of another desirable conformal array. The approach is demonstrated by numerical experiments including a full wave simulation.**

***Index Terms*—Metamaterial, Lenses, Quasi-Conformal Optical Transformation, Conformall Array**

I. Introduction

Conformal arrays have vast areas of applications especially in airborne systems and satellites [1]-[3]. Nevertheless, synthesis of conformal arrays has always been a challenging task mainly because the complexity of the conformal structure and the fact that radiating elements are pointed in different directions [4]. Several techniques have been proposed in the literature to address this problem, including techniques based on projection methods [5]-[7], iterative least square techniques [8][9] and various global optimization techniques [10]-[13]. More recently, solutions based on the Bayesian Compressive Sampling method have been presented as well [14].

An alternative approach to synthesis of conformal arrays is to use Metamaterial Lenses. Driven by formulations of Transformation Optics [15]-[18] and the pursuit of realization of material properties which cannot be readily found in nature [19]-[21], solutions based on metamaterials have flourished. Among these solutions, several types of lenses, which have been designed to manipulate radiation properties of antennas, have been proposed. More in detail, a class of transformations identified as Quasi-Conformal Transformation Optics (QCTO) was proposed to ease material property requirements in [22]. The QCTO method is known for its ability to mitigate the anisotropic nature of lenses and therefore to ease of their simulation and fabrication [22][23]. Such a feature is motivated by the fct that QCTO designs can be numerically obtained by solving Laplace's equation with sliding boundary condition, which guarantees "smooth" features to the arising transformation grid [23]-[25].

QCTO has been used in [24] to transform an array with an arbitrary shape to a linear array and vice versa. This led to the design of linear arrays exhibiting properties of conformal arrays [24]. In this paper, the approach proposed in [24][26] is extended to a multi-step quasi-conformal transformation and is used to design lenses that enhance performance of conformal arrays. The lens allows the conformal array to behave like another arbitrary-shaped array, for instance a circular array where the radiation pattern is stable when varying the steering angle.

The main advantage of using metamaterial lenses in a conformal array with respect to the traditional synthesis methods is the ability to transform existing well-investigated conformal array synthesis techniques, such as those for circular or spherical arrays, in potentially arbitrary geometry configurations.

II. Multi-step transformation

The multi-step transformation employed in this paper involves the cascaded combination of ordinary transformations. In [24][25] it has been shown that QCTO can be used to transform a region conformal to an array geometry to a flat rectangular region. In this paper, two such transformations are cascaded to enable transformation between two conformal regions. The extension of formulations of the single step transformation to a multi-step is straight forward. Considering a 2D scenario, if $\hat{\Gamma}$ represents a primary transformation $(x'', y'') \rightarrow (x', y')$ and $\tilde{\Gamma}$ represents a secondary transformation $(x', y') \rightarrow (x, y)$, the Jacobian matrices of the two transformations can be written as

$$\underline{\hat{\Lambda}} = \begin{bmatrix} \frac{\Delta x'}{\Delta x''} & \frac{\Delta x'}{\Delta y''} & 0 \\ \frac{\Delta y'}{\Delta x''} & \frac{\Delta y'}{\Delta y''} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \underline{\tilde{\Lambda}} = \begin{bmatrix} \frac{\Delta x}{\partial x'} & \frac{\Delta x}{\Delta y'} & 0 \\ \frac{\Delta y}{\Delta x'} & \frac{\Delta y}{\Delta y'} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

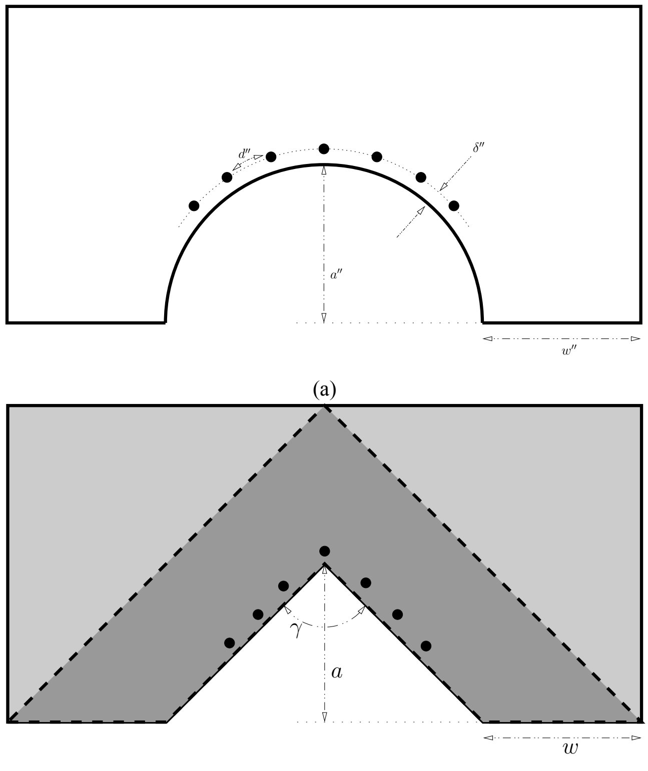
Hence, the relative permittivity in the corresponding mediums can be related as [25]: $\underline{\varepsilon}' = \frac{\underline{\hat{\Lambda}}\,\underline{\varepsilon}''\,\underline{\hat{\Lambda}}^T}{\det(\underline{\hat{\Lambda}})}$ and $\underline{\varepsilon} = \frac{\underline{\tilde{\Lambda}}\,\underline{\varepsilon}'\,\underline{\tilde{\Lambda}}^T}{\det(\underline{\tilde{\Lambda}})}$, where $\underline{\varepsilon}''$, $\underline{\varepsilon}'$ and $\underline{\varepsilon}$ are relative permittivity tensors in three spaces (x'', y''), (x', y') and (x, y) respectively. Merging the last two equations and doing similar analysis for the relative

This work has been developed within the EMERALD project funded by the Autonomous Province of Trento – Calls for proposal "Team 2011".

permeability tensors, the following relationships can be established.

$$\underline{\varepsilon} = \frac{\left(\underline{\tilde{\Lambda}}\,\underline{\hat{\Lambda}}\right)\underline{\varepsilon}''\left(\underline{\tilde{\Lambda}}\,\underline{\hat{\Lambda}}\right)^T}{\det\left(\underline{\tilde{\Lambda}}\,\underline{\hat{\Lambda}}\right)},\quad \underline{\mu} = \frac{\left(\underline{\tilde{\Lambda}}\,\underline{\hat{\Lambda}}\right)\underline{\mu}''\left(\underline{\tilde{\Lambda}}\,\underline{\hat{\Lambda}}\right)^T}{\det\left(\underline{\tilde{\Lambda}}\,\underline{\hat{\Lambda}}\right)} \qquad (2)$$

The individual transformations $(x'', y'') \to (x', y')$ and $(x', y') \to (x, y)$ are made using the numerical transformation method discussed in [24][25].



(b)

Fig. 1. Transformation regions: Layout of (a) Circular virtual array and (b) Physical Array mounted on sharp corner.

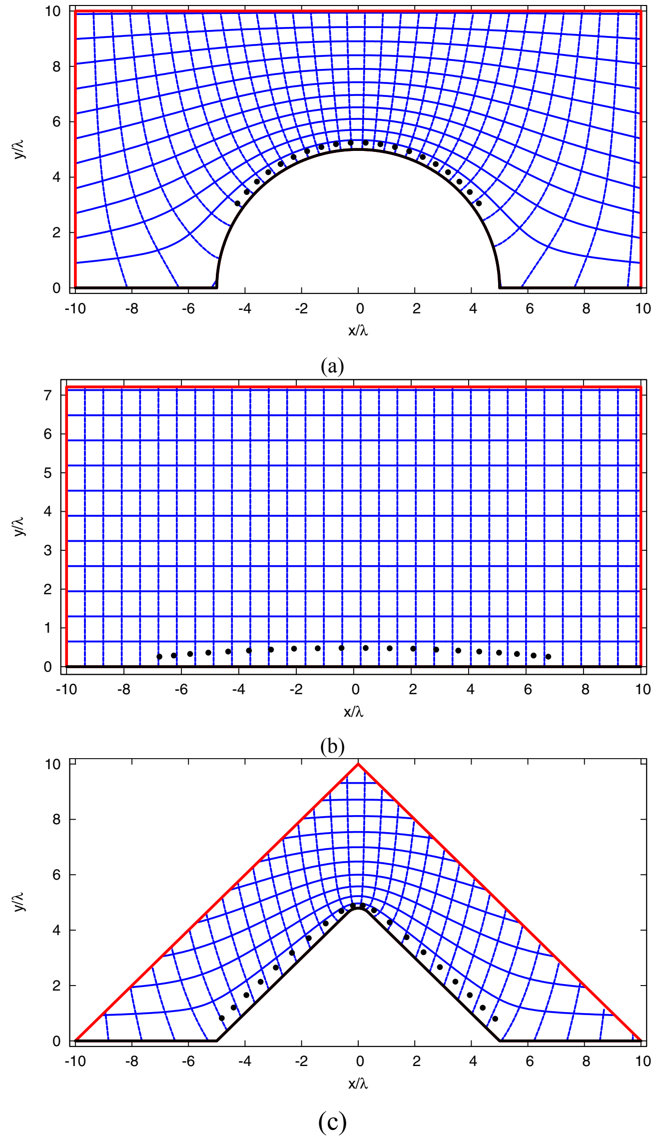
III. Numerical Results

To demonstrate the proposed methodology, synthesis of an array mounted on a sharp corner is presented here. The array is coated with a metamaterial lens to enable the formation of a stable pattern as that would be generated from a circular array of equivalent dimensions.

Fig. 1 shows the transformation region pairs; in Fig. 1(a) the characterization of the virtual or target array configuration is presented. The goal is to synthesize an array with the properties of a circular configuration. The elements of the actual array are to be distributed on the sharp corner [Fig. 1(b)]. The rectangular region shown in Fig. 1(a) is quasi-conformally mapped to the outer rectangular lightly shaded region in Fig. 1(b) using a two step numerical transformation. The exterior parts of transformed region in Fig. 1(b) are then trimmed to leave a sharp edged lens which is conformal to the intended array geometry without significantly affecting the performance of the lens. The actual lens is the internal part in Fig 1(b) contained inside the broken line.

The dimensions of the transformation regions referring to Fig. 1 are: Number of array elements $N = 20$ operating at frequency of $\nu = 600MHz$; $a = a'' = 5\lambda$; $\omega = \omega'' = 5\lambda$; $d'' = 2\delta'' = \lambda/2$; $\gamma = 90^0$, where λ is the wavelength.

The array in virtual space is uniformly excited with its main beam oriented towards $\phi_s = 60^0$; the phase of the excitation is computed as $\varphi_n = \left(-2\pi a''/\lambda\right)\cos(\phi_s - \phi''_n)$, where ϕ''_n is the angular position of the n^{th} element.



(c)

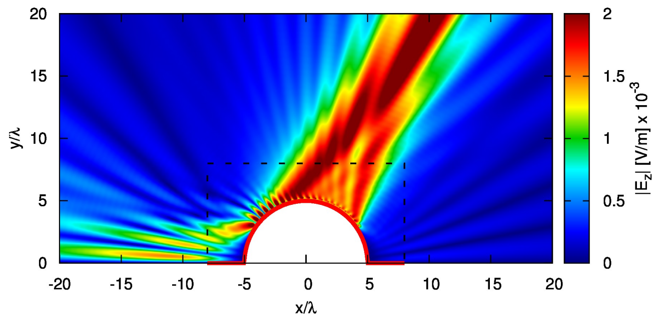
Fig. 2. Transformation Grid: Grids in (a) virtual space, (b) intermediate space, and (c) actual physical lens.

The quasi-orthogonal grids in the two conformal regions were generated following the procedures in [24][25]. Fig. 2(a) shows the grids in the virtual space where there is a circular array radiating in a free space ($\underline{\varepsilon}'' = \underline{\mu}'' = I$; I is the identity matrix.). For the grid in the physical space [Fig. 2(c)], a small fillet of radius $\rho = 0.25\lambda$ is introduced to avoid singularities during grid generation.

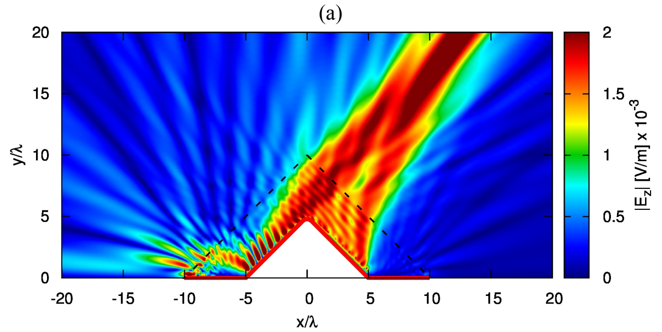
The permittivity and permeability of the physical metamaterial lens ($\underline{\varepsilon}$ and $\underline{\mu}$) are computed using (1) and (2); and a 2D full-wave simulation was performed using COMSOL multiphysics. The outputs of the simulation are shown in Fig. 3. In Fig. 3(a) the magnitude of the normal component of the electric field radiated from the target circular array is presented and Fig. 3(b) shows the equivalent field distribution generated by the array mounted on a sharp corner and coated with the metamaterial lens. The two field distributions have good agreement outside the lens.

IV. CONCUSIONS

A multi-step quasi-conformal transformation approach has been used to design a lens to enhance the radiation properties of a conformal array. The approach is a flexible one that can support transformation between two conformal arrays configurations.



(a)



(b)

Fig. 3. Simulation Results: Magnitude of Electric field radiated from (a) circular array, array mounted on sharp corner with metamaterial lens coating.

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