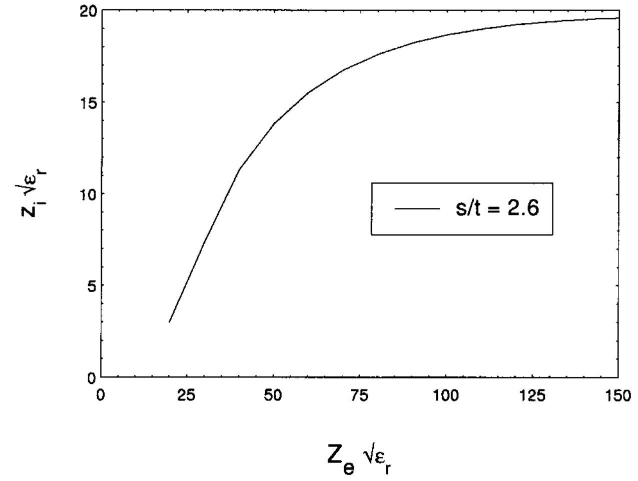

Figure 3 Maximum value of the image impedance as a function of the odd-mode characteristic impedance for $s/t = 2.6$

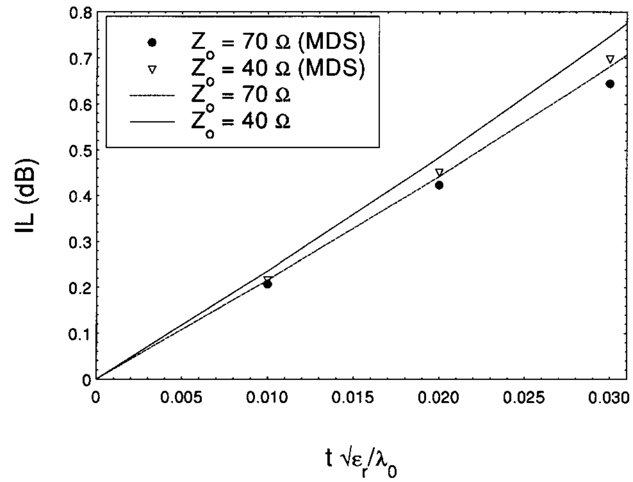

Figure 4 Insertion loss (IL) of the coupler as a function of the normalized strip-line thickness

wave moment method calculations [4]. The agreement between the results is remarkably good.

Figure 4 can be used to guide the coupler design. After the selection of the frequency, substrate material (ϵ_r), and determination of the maximum allowable loss, the appropriate maximum substrate thickness is found from Figure 4. The aperture width is calculated using the fact that $s/t = 2.6$. Since the IL is almost independent of Z_o, the designer has the freedom to choose its value, thereby also fixing Z_i. This step can be guided by Figure 3.

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POLARIZATION-SELECTIVE SURFACES COMPOSED OF TREFOIL KNOT ELEMENTS

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ABSTRACT: *In this paper, we demonstrate that the trefoil knot has the unique property that its forward scattering for circularly polarized waves exhibits a strong dependence on the handedness of the polarization. Both a single knot element as well as an array of these knots show this property which could be used in a polarization-selective surface (PSS).* © 1999 John Wiley & Sons, Inc. Microwave Opt Technol Lett 21: 170–173, 1999.

Key words: *electromagnetic scattering; polarization; polarization-selective surface; knot electrodynamics*

1. INTRODUCTION

Knot electrodynamics is a relatively new area of research which seeks to combine aspects of knot theory with Maxwell's theory of electromagnetism. The electromagnetic radiation and scattering properties of thin perfectly conducting, toroidally knotted wires have recently been investigated in [1, 2]. The so-called (p, q)-torus knots represent a class of knots which reside on the surface of a solid torus with longitudinal radius a and cross-sectional radius b. These knots are classified by the relatively prime integers p and q, where p represents the number of turns around the torus in the longitudinal direction, and q is the number of turns around the torus in the meridional direction. A useful set of parametric representations for the (p, q)-torus knots is [1, 2]

$$x = (a + b\cos(\psi + qs))\cos(ps) \tag{1}$$

$$y = (a + b\cos(\psi + qs))\sin(ps) \tag{2}$$

$$z = b\sin(\psi + qs) \tag{3}$$

where $0 \le s \le 2\pi$ and, without loss of generality, we may set $\psi = 0$. Note that for fixed values of a, b, p, and q, the parameterizations (1)–(3) will trace out the (x, y, z)-coordinates of the associated torus knot as s varies from 0 to 2π. These parameterizations may also be used to derive a useful formula for the arc length of a (p, q)-torus knot given by [1]

$$L_{pq} = 2\int_0^{\pi} \sqrt{p^2(a + b\cos u)^2 + (qb)^2}\, du. \tag{4}$$

In this letter, attention will be focused on the simplest nontrivial torus knot known as the trefoil, viz. the (2, 3)-torus knot. Two views of this trefoil are shown in Figure 1. The trefoil belongs to the class of knots which have both geometrical and topological chirality [3]; in other words, the trefoil is

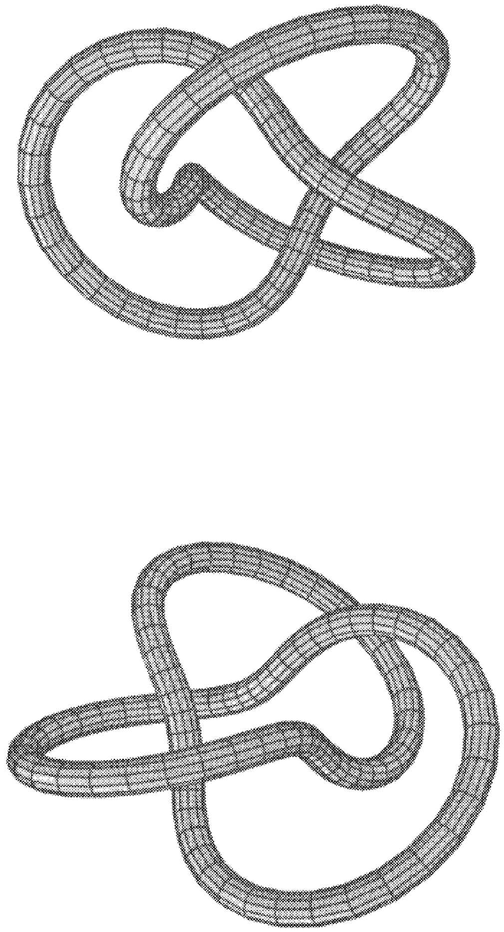


Figure 1 Two views of a (2, 3)-torus knot (the trefoil)

distinct from its mirror image in the geometrical sense as well as the topological sense. This fact suggests that there are actually two different knots, a right-handed and a left-handed trefoil. It will be demonstrated here that this property plays a key role in the design of a right-hand or left-hand polarization-selective surface (PSS).

In contrast to the basic wire loop which only couples to the axial H-field, it has been shown that the trefoil knot responds to electromagnetic fields incident at an arbitrary angle [1]. Furthermore, the knot is only weakly depolarizing in the backscatter direction over a wide range of frequencies. In this paper, we consider circularly polarized incident plane waves, which are traveling in the negative z-direction, and whose major axis is in the positive x-direction. The forward and backscattering characteristics of the knot are computed via the method of moments for the single knot as well as for a 5×5 array of identically oriented trefoil knots.

2. SINGLE TREFOIL

In this section, we consider the scattering properties of a single trefoil that has an arc length of 1.5 m. In Figure 2, we plot the forward and backscattered far fields (excluding the factor e^{-jkr}/r) for both left- and right-handed circularly polarized incident plane waves. The first resonance, which occurs at approximately 200 MHz, corresponds to when the arc length of the trefoil is on the order of a wavelength and is seen to be very sharp.

The backscatter characteristics of this trefoil were found to be relatively insensitive to both the incidence angle as well as the polarization of the incident field. The forward characteristics of the knot are exhibited by the dashed lines in Figure 2. For frequencies below the first resonance, the left circularly polarized wave always dominates in the forward scatter direction. At resonance, the forward scatter is almost five times higher than the backscatter. Once we go past this resonance to higher frequencies, the forward scattered field drops below the backscattered field, and continues in this

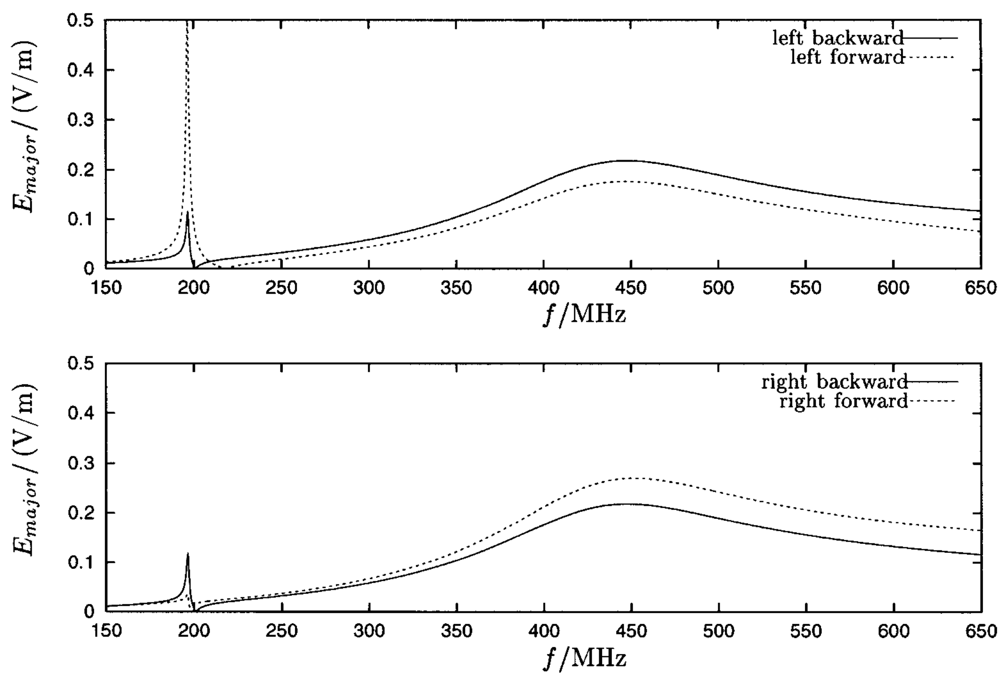
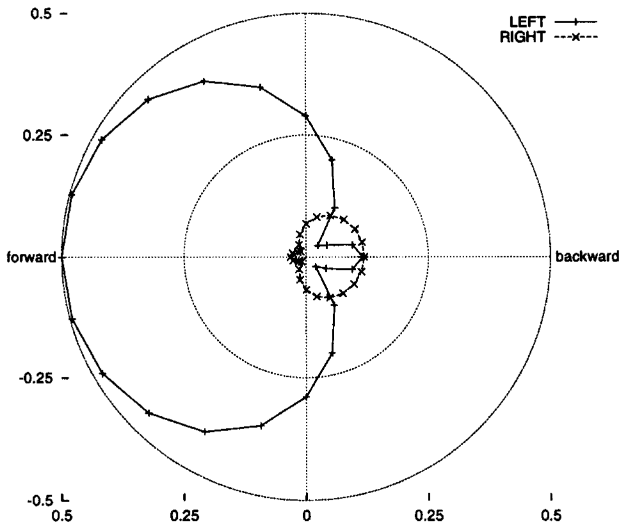


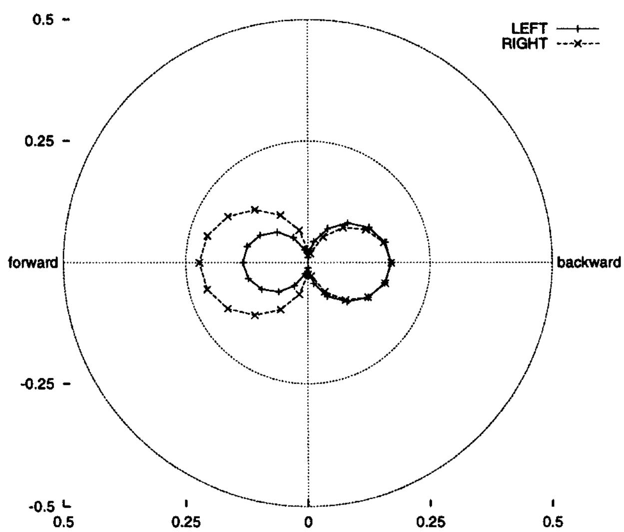
Figure 2 Forward and backscatter characteristics of a knot for right and left circularly polarized incident waves

position over the entire high-frequency range. In contrast, for the right circularly polarized wave, the forward scattered field is always lower than the one in the backscatter direction below the resonance. At the first resonance, the forward scattered field is almost five times smaller than the corresponding backscattered field. However, the two curves cross over after the first resonance, and the forward scattering dominates over the remainder of the frequency range.

In Figure 3, we show a polar plot of the far field for two frequencies, viz. $f = 196$ MHz for the graph on the top [Fig. 3(a)] and $f = 447$ MHz for the bottom [Fig. 3(b)]. Both correspond to a local maximum of the backscatter within the frequency range of 150–650 MHz. The diagram for the left-hand polarized wave, marked with "+," clearly shows that for $f = 196$ MHz, the forward scatter is much higher than the scatter in the backward direction. The right-hand polarized wave, designated by "×," scatters almost entirely in the backward direction. The bottom graph shows the radiation pattern for $f = 447$ MHz. As mentioned earlier, the forward scatter is now lower than the backscatter for the left-hand polarized wave and vice versa for the right-hand polarized wave. The remarkable scattering properties of the trefoil knot demonstrated in Figure 3 will be exploited below in order to develop a design methodology for polarization-selective surfaces capable of responding to the handedness of incident circularly polarized waves.



(a)



(b)

Figure 3 Radiation patterns for (a) $f = 196$ MHz and (b) $f = 447$ MHz, indicated by "+" for the left circularly polarized wave, and "×" for the right circularly polarized wave

3. ARRAY OF TREFOILS

An analysis of an array of the above knots reveals that the basic scattering characteristics of the single knot are preserved. To illustrate this property, we investigate a 5×5 array of knots with a center-to-center spacing of 0.5 m as shown in Figure 4. This particular spacing is chosen because the knots are only weakly coupled to each other, as will be shown in an upcoming paper by the authors. Due to this weak interaction between the individual knots, the scattering property of the array essentially represents an aggregate of the contributions of individual elements, added up in phase, both in the forward and the backscatter directions. The forward and backscattered far-field characteristics of the array, displayed in Figure 5, confirm the above observation. Hence, we conclude from this study that the array effectively acts as a polarization-selective filter. We also note that if we choose the trefoil knot of opposite handedness as the PSS array element, then the PSS will correspondingly respond to the opposite polarization.

4. CONCLUSIONS

The forward scattered fields from a single, as well as from an n-element array of trefoil knots, exhibit a very strong dependence on the polarization of the incident wave. In contrast, the backscattered field is independent of the polarization, and hence, of the handedness of the incident wave. The trefoil knot could therefore find application in the design of a polarization-selective filter for circularly polarized waves, especially when operated at resonance.

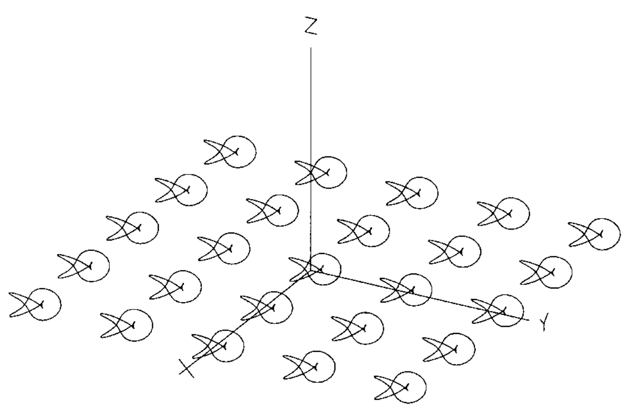


Figure 4 5×5 array of trefoil knots

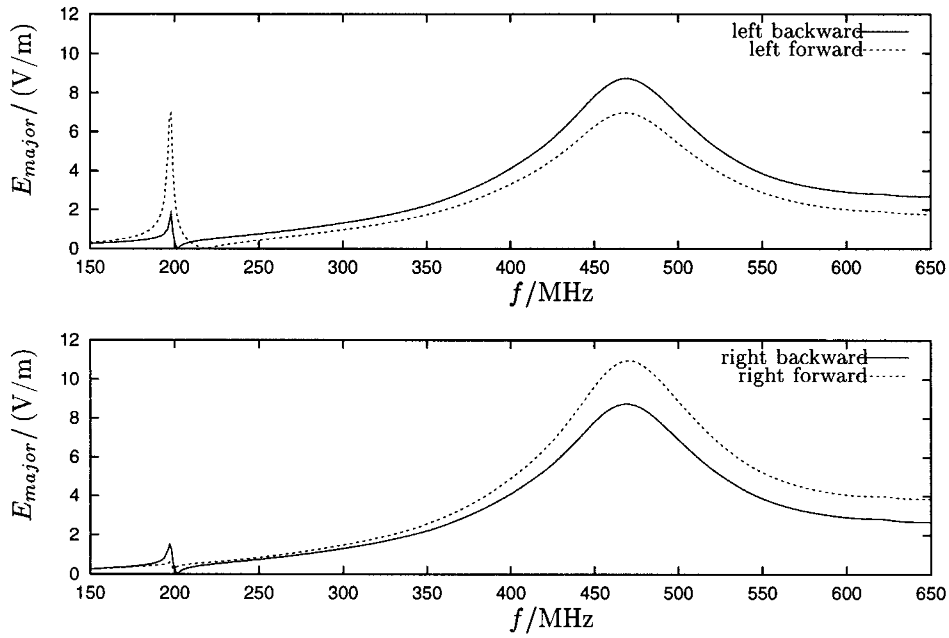


Figure 5 Forward and backscatter characteristics of a 5 × 5 array of identical trefoil knots for left and right circularly polarized incident waves

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PO EVALUATION OF THE FIELD BACKSCATTERED BY A TRIHEDRAL CORNER WITH A RESISTIVE FACE

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ABSTRACT: *The field backscattered by a resistive trihedral corner is evaluated by using a physical optics model. This modified trihedral is obtained from the conventional one by substituting a metallic plate with a dielectric sheet, and presents a polarization response which makes it particularly attractive to be used in polarimetric radar systems.* © 1999 John Wiley & Sons, Inc. Microwave Opt Technol Lett 21: 173–177, 1999.

Key words: *electromagnetic scattering; resistive trihedral corner reflector; physical optics*

1. INTRODUCTION

A conventional triangular trihedral corner reflector (TCR) consists of three mutually orthogonal metal plates. Besides exhibiting a large monostatic radar cross section over a wide angular range, it is a passive device with low manufacturing costs, and the maintenance of its efficiency is simple and not expensive. These characteristics have justified its diffusion as a radar enhancement device and location marker for navigational purposes [1], and as ground external calibrator for radar systems. In particular, in microwave remote sensing, there is a well-assessed use of TCRs for the calibration of synthetic aperture radar (SAR) images [2]. Such a TCR is suitable for linearly polarized radars since it returns linearly polarized incident electromagnetic waves without modification. On the contrary, it reverses the rotation sense of circularly polarized incident waves. On the other hand, with the use of circularly polarized radars, it becomes very important to have a TCR designed to return circularly polarized waves with the same handedness as the incident waves.

A physical optics (PO) model to evaluate the field backscattered by a resistive TCR (see Fig. 1) is developed in this work. This modified TCR is obtained from the conventional one by substituting a metallic plate with a dielectric sheet as proposed by Kennaugh in [3] in order to return a circularly polarized incident wave without changing the sense of rotation. In fact, the presence of a dielectric plate modifies the polarization response since the relative phase of parallel and perpendicular components of the wave partially reflected by a dielectric interface can differ remarkably from that obtained at a metallic wall.

The model proposed here uses geometrical optics (GO) to determine in analytic form the illuminating electric field related to the direct, singly, and doubly reflected rays incident on each TCR face and the shape of the corresponding